Single Mechanics – Vector Motion 2

Questions taken from *January* Edexcel papers up to (and including) 2010.

1. A particle P moves with constant acceleration $(2\mathbf{i} - 5\mathbf{j})$ ms^{-2}. At time $t = 0$, P has speed u ms^{-1}. At time $t = 3$ s, P has velocity $(-6\mathbf{i} + \mathbf{j})$ ms^{-1}.

   Find the value of u. [20]

2. Two boats A and B are moving with constant velocities. Boat A moves with velocity $9\mathbf{j}$ kmh^{-1}. Boat B moves moves with velocity $(3\mathbf{i} + 5\mathbf{j})$ kmh^{-1}. At noon, A is at point O, and B is 10 km due west of O. At time t hours after noon, the position vectors of A and B relative to O are $\mathbf{a}$ km and $\mathbf{b}$ km respectively.

   (a) Find the bearing on which B is moving. □

   (b) Find expressions for $\mathbf{a}$ and $\mathbf{b}$ in terms of t, giving your answer in the form $p\mathbf{i} + q\mathbf{j}$. □

   (c) Find the time when B is due south of A. □

   (d) At time t hours after noon, the distance between A and B is d km. By finding an expression for $\overrightarrow{AB}$, show that $d^2 = 25t^2 - 60t + 100$. □

   (e) At noon, the boats are 10 km apart. Find the time after noon at which the boats are again 10 km apart. □

3. Two ships P and Q are travelling at night with constant velocities. At midnight, P is at the point with position vector $(20\mathbf{i} + 10\mathbf{j})$ km relative to a fixed origin O. At the same time, Q is at the point with position vector $(14\mathbf{i} - 6\mathbf{j})$ km. Three hours later, P is at the point with position vector $(29\mathbf{i} + 34\mathbf{j})$ km. The ship Q travels with velocity $12\mathbf{j}$ kmh^{-1}. At time t hours after midnight, the position vectors of P and Q are $\mathbf{p}$ km and $\mathbf{q}$ km respectively.

   (a) Find the velocity of P, in terms of $\mathbf{i}$ and $\mathbf{j}$. □

   (b) Find expressions for $\mathbf{p}$ and $\mathbf{q}$, in terms of t, $\mathbf{i}$ and $\mathbf{j}$. □

   (c) At time t hours after midnight, the distance between P and Q is d km. By finding an expression for $\overrightarrow{PQ}$, show that

   $$d^2 = 25t^2 - 92t + 292.$$

   (d) Weather conditions are such that an observer on P can only see the lights on Q when the distance between P and Q is 15 km or less. Given that when $t = 1$, the lights on Q move into the sight of the observer, find the time, to the nearest minute, at which the lights on Q move out of sight of the observer. □